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Real Options, Preemption, and the Dynamics of Industry Investments *

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Abstract

We study the development of a duopoly industry −evolution of firm capacities and competitive behavior− in a continuous-time real-options model of capacity investment. Our methodology allows the evaluation of investment options and exercise rules in a strategic setup. In the initial industry development phase, firms attempt to preempt each other, so that the first industry investment occurs earlier than socially optimal and the first entrant takes more risk than socially optimal. While capacity units are costly, indivisible, durable, and big relative to market size, early entry cannot secure a first-mover advantage, so that both firms are active beyond some level of market development. Once both firms hold capacity, tacit collusion, taking the form of postponed capacity investment, may occur in Markov Perfect Equilibrium. Volatility and the expected speed of market development play a crucial role in the determination of competitive behavior: we show that a tacit-collusion equilibrium is certain to exist when market growth is highly volatile and/or very fast.

key words: Real options; Option value; Duopoly; Preemption; Collusion; Capacity; Industry growth; Volatility; Risk.

J.E.L. classification: C73, D43, D92; L13.

Résumé

Nous étudions le développement d'une industrie −capacités et comportement concurrentiel− dans un modèle en temps continu d'options réelles d'investissement en capacité. Notre méthodologie permet l'évaluation des options et des règles d'exercice en contexte stratégique. Initialement, les firmes ont un comportement de préemption, si bien que le premier investissement en capacité se produit plus tôt, et comporte un risque plus élevé, que socialement désirable. Bien que les unités de capacité soient coûteuses, indivisibles, durables et de taille non négligeable par rapport au marché, l'entrée hâtive ne peut conférer d'avantage durable; à partir d'un certain niveau de développement du marché, les deux firmes sont en activité. Alors, une collusion tacite pour retarder les augmentations de capacité subséquentes peut devenir possible en équilibre Markovien parfait. La volatilité du marché et sa vitesse de croissance jouent ici un rôle crucial: l'équilibre de collusion tacite existe si la croissance est très volatile et/ou très rapide.

mots-clés: Options réelles; Valeur d'option; Duopole; Préemption; Collusion; Capacité; Croissance du marché; Volatilité; Risque.

Classification J.E.L.: C73, D43, D92; L13.

1. Introduction

Among investment decision tools, real option theory is reaching advanced textbook status and is rapidly gaining reputation and influence. Although both popular writers and specialists warn against its often daunting complexity, they also stress its unique ability to take account of future flexibility and the importance of future moves and decisions in valuing current investments. The real options approach emphasizes the indivisibility and irreversibility of investments; indivisibilities often imply a limited number of players, hence imperfect competition. Yet, while it is often stressed that real option theory is best to analyze investments of strategic importance – the word 'strategic' appears repeatedly in the real-options literature – the bulk of that literature involves decision makers playing against nature rather than against other players. The analysis of strategic considerations, in a game theoretic sense, is still in its infancy and should be high in the real-option research agenda. Notable exceptions are Grenadier (1996) who uses a game-theoretic approach to option exercise in the real estate market; Smets (1995) who provides a treatment of the duopoly in a multinational setup, which serves as a basis for the oligopoly discussion in Dixit and Pindyck (1994, pp. 309-14); Lambrecht and Perraudin (1996) and Mariotti and Décamps (2000) who investigate the impact of asymmetric information about costs on firms' investment strategies; and Weeds (1999) who study option games in a technology adoption context.

Our paper extends these pioneering contributions while bringing to bear the older, and highly relevant, literature on strategic investment, most notably Gilbert and Harris (1984), Fudenberg and Tirole (1985), and Mills (1988). These papers help address surprisingly modern questions at the dawn of the information technology revolution: what is the role of investment decisions in shaping the structure of a developing sector? Do competing investments lead to preemption, rent equalization, and rent dissipation as in Fudenberg and Tirole? Or are firms able to tacitly collude in a non cooperative way and avoid cannibalism despite the threat of preemption? Can a first-mover advantage be maintained and reinforced as in Gilbert and Harris (1984) or Mills (1988) or does the laggard catch up? In which ways are option values and exercise rules affected by such strategic considerations? The recent synthetic work of Athey and Schmutzler (2001) brings more generality and clarity to our understanding of the role of investment in market dominance. They provide conditions on current payoffs for weak increasing dominance, in a framework that encompasses as special cases such models as Bertrand or Cournot competition with differentiated goods, horizontal competition on the line, and vertical quality differentiation. However, they also show that, when firms are farsighted and are not forced to commit to strategic investment plans in advance, there is little hope to obtain definitive predictions outside more specific models. This is precisely the setup considered in our paper: dynamic investment without commitment, Markov perfect strategies. We restrict our attention to duopoly on a homogeneous product market with incremental indivisible capacity investments, while paying particular attention to the role of uncertainty and the speed of market development on investment strategies and competition.

From a methodological point of view, the paper uses the formalism of real options: we find optimal exercise rules and we evaluate the corresponding options. However these options correspond to the payoffs of particular strategies in a game-theoretic sense. In order to investigate the above game-theoretic issues in a real-options framework, we need to address them in a continuous-time context where irreversible investment decisions are made by rival firms under uncertainty about the future evolution of the market and the industry. We achieve this by extending Fudenberg and Tirole (1985)'s formalism for modelling games of timing to such an environment.

While the basic economic model used throughout the article is very similar to Gilbert and Harris' (an industry faces growing demand with indivisibilities in installing new capacity; firms have access to the same technology; time is continuous), using these more recent contributions allows us to avoid any technical assumption that gives a first-mover advantage to a player. Since we want to investigate preemption and other strategic aspects, we assume that the firms cannot commit *ex ante* to any sequence of investments. Our analysis is restricted to duopoly.

We show that both the size of capacity units relative to the market, and the relative existing capacities of the firms are important in their own way. In our model, market develops indefinitely, but the basic unit of capacity never becomes negligible relative to market size. Yet excess capacity cannot be used by one firm to hold the other one at bay permanently. If one firm holds excess capacity, the other firm will eventually hold enough capacity to serve half the market. This is in sharp contrast with Gilbert and Harris' famous preemption equilibrium where a single firm accounts for the totality of industry capacity, although without enjoying any more profits than its dwarfed rivals. If both firms are restricted to one more investment at most, a setup similar in that respect to Fudenberg and Tirole (1985), Grenadier (1996), Weeds (1999), and others, we show that the smaller firm moves first in a preemption equilibrium.

As other authors have already found in related models, two types of equilibria may arise: preemption equilibria involving rent equalization and dissipation, and equilibria involving tacit collusion. Although collusion equilibria do not necessarily maximize joint profits, they are Pareto superior to preemption equilibria from the firms' point of view as the firms implicitly agree to postpone their investment in such a way as to preserve existing rents.

Low initial capacities are of particular interest in the case of emerging sectors. When a firm does not hold any existing capacity it cannot be threatened with the loss of any existing rent; as a result a tacit-collusion equilibrium cannot be enforced and preemption is the sole equilibrium. Thus the initial development of an industry is highly competitive although the preemption equilibrium is characterized by the presence of only one active firm at first. Paradoxically, once both firms are active tacit-collusion equilibria may be possible so that the industry may become less competitive despite the presence of more active firms. Collusion is also more efficient between firms of equal sizes in the sense that, when collusion equilibria exist, the joint investment date that maximizes combined profits is an equilibrium; in contrast collusive strategies that maximize combined profits do not yield an equilibrium when firms are not of equal size.

It is well known that higher volatility raises the value of investment (call) options because the decision maker can achieve higher exposition to upside movements while being protected from downside ones. In a strategic setup volatility further affects collusion opportunities. More precisely we find that above some threshold level of uncertainty, collusion equilibria always exist among firms that hold positive capacity. The speed of market development plays a role similar to the drift of the underlying asset in financial options. Under usual assumptions, the drift does not affect the value of a financial option; volatility alone matters. However such result does not obtain here; market growth affects investment option values, together with volatility and other parameters. This is because, in a non perfectly competitive context, we cannot adopt the spanning assumption frequently made in financial and real options analyses (see Dixit and Pindyck, 1994) and which make expected capital gains on the underlying asset irrelevant. Moreover, we show that in such a context of strategic real options, market growth can affect collusion opportunities: there is an expected market growth rate above which tacit-collusion equilibria exist.

While a general characterization of the solution is quite involved, considering a succession of special cases will bring up the issues and mechanisms involved. This will highlight the important role played by capacity acquisitions and existing capacity, and by the volatility and speed of the market growth process.

After presenting the model and its short-run properties in Section 2, we develop the corresponding real-options valuation functions in Section 3 and we define the corresponding concepts of Markov strategies and equilibrium. We then proceed with a sequence of sections focusing on particular issues, starting in Section 4 with a game where firms invest at most once and do not hold any initial capacity, continuing in Section 5 with a game where firms initially hold one capacity unit each and may invest once, then allowing initial capacities to differ in Section 6. These particular cases allow us to establish the role of existing capacity and capacity differences on preemption and collusion, and to show how uncertainty and market growth affect strategic possibilities and outcomes in a real-options context.

2. Model and short-run aspects

2.1. Market demand and firms

We consider the development of a duopoly industry where demand can change unpredictably because of random aggregate shocks. Time is continuous, and indexed by $t \in [0, \infty)$. The inverse demand function at time $t \geq 0$ is given by:

$$P(t, X_t) = Y_t \, D^{-1}(X_t), \tag{1}$$

where $X_t \geq 0$ is aggregate output, assumed to be produced from the productive assets in place in the industry, $Y_t \geq 0$ is an industry-wide random shock, and $D : \mathbb{R}_+ \to \mathbb{R}_+$ is a mapping describing the non-stochastic component of the market demand curve. Unless otherwise stated, we make the following assumptions about D:

Assumption 1

D1. D is strictly decreasing and $D(0) = \lim_{p \downarrow 0} D(p) < \infty$;

D2. D is continuously differentiable and integrable on $\mathbb{R}_+$;

D3. The mapping $x \mapsto xD^{-1}(x)$ is strictly concave on $(0, D(0))$.

Note that the aggregate shocks $(Y_t)_{t \geq 0}$ only affect the willingness to pay of the consumers for any given quantity of the good. In particular, the maximal capacity of the market, $D(0)$, is fixed and independent from the current shock.[1] Uncertainty is represented by a complete probability space $(\Omega, \mathcal{F}, P)$. The stochastic process of aggregate

[1] Thus market demand is driven by consumers' tastes for the output, and not by replication of the initial consumers as in Gilbert and Harris (1984).

shocks $(Y_t)_{t\geq 0}$ is modeled as a geometric Brownian motion:

$$dY_t = \alpha Y_t\, dt + \sigma Y_t\, dZ_t, \quad Y_0 > 0, \tag{2}$$

where $\alpha, \sigma > 0$ and $(Z_t)_{t\geq 0}$ is a standard Brownian motion with respect to $(\Omega, \mathcal{F}, P)$. Thus the flow of information accruing to all agents is represented by the complete continuous filtration $(\mathcal{F}_t)_{t\geq 0} = (\sigma\{Y_s \,|\, s \leq t\})_{t\geq 0}$ generated by $(Y_t)_{t\geq 0}$.

Firms are risk neutral and discount future revenues at a risk free rate $r > \alpha$.[2] Variable costs are nil. Together with (1), this implies that, absent any capacity constraint, industry output is independent of Y_t. Investment takes place in a lumpy way. The cost of each capacity unit is I in current value and is constant over time. Each capacity unit produces at most one unit of output; although it retains its productive capacity forever, it has no resale value. Unless otherwise mentioned, each firm may hold any arbitrary number of capacity units. We now investigate what happens if the industry is a duopoly.

2.2. The short-run duopoly game

Firm i's capacity at t, its number of installed capacity units, is denoted K^i_t. The short run is characterized by Cournot competition over quantities, subject to capacity constraints. The instantaneous level of prices is determined by aggregate output and industry-wide shocks through the inverse demand function. Let $X^i \leq K^i$ be firm i's output, and let $\Pi(X^i, X^{-i}, Y_t)$ denote firm i's current profit given that the other firm $-i$, produces X^{-i}. The maximization of $\Pi(X^i, X^{-i}, Y_t) = Y_t X^i D^{-1}(X^i + X^{-i})$ requires that X^i be chosen such that:

$$X^i D^{-1\prime}(X^i + X^{-i}) + D^{-1}(X^i + X^{-i}) \geq 0,$$

with equality if $X^i < K^i$. This defines i's reaction function, which is independent of Y_t. Three possible equilibria of the (short-run) stage game in quantities may emerge,

[2] If $\alpha \geq r$, each firm's value would be maximized by perpetually holding its investment options alive, and investment would never take place.

according to which constraints are binding or not:

$$\begin{aligned} X^i = K^i \quad &\text{and} \quad X^{-i} = K^{-i}, \\ X^i D^{-1\prime}(X^i + X^{-i}) + D^{-1}(X^i + X^{-i}) = 0 \quad &\text{and} \quad X^{-i} = K^{-i}, \\ x^c D^{-1\prime}(2x^c) + D^{-1}(2x^c) = 0 \quad &\text{and} \quad X^i = X^{-i} \equiv x^c, \end{aligned} \tag{3}$$

where x^c is the unconstrained Cournot output as defined by (3).

Thus in the short-run equilibrium each firm i's output is a function $X\left(K^i, K^{-i}\right)$ of the capacities of both firms. Note that if only one firm, say firm i, is constrained, it must be the smaller firm: $K^i < K^{-i}$. Note also that no firm will ever need more than m units, the minimum capacity at which a monopoly is unconstrained. We will denote π_{kl} the short-run instantaneous profit, divided by Y_t, of a firm with capacity k when the other firm holds l capacity units: $\pi_{kl} = \frac{1}{Y_t}\Pi(X\left(k,l\right), X\left(l,k\right), Y_t) = X\left(k,l\right) D^{-1}(X\left(k,l\right) + X\left(l,k\right))$. To illustrate, suppose that $D^{-1}\left(X\right) = 1 - X/6$, and that one firm holds three capacity units while its opponent holds one unit. In the short-run equilibrium, the small firm produces at full capacity while the bigger one produces $\frac{5}{2}$ for an instantaneous profit of $Y_t \pi_{31}$ where $\pi_{31} = \frac{25}{24}$; similarly $\pi_{13} = \frac{5}{12}$.

3. The long-run

3.1. Investments and firm valuation

Since $(Y_t)_{t \geq 0}$ is a time homogenous Markov process, any outcome may be described as an ordered sequence of investment triggers, together with the short-run instantaneous profits induced by each investment for both firms between investments. We note y_{ij} the investment triggers, where i is the capacity, immediately before Y_t reaches y_{ij} for the first time, of the firm that initiates the investment (the leader), and j is the capacity of its opponent. Thus if both firms hold no capacity and one firm starts with an investment of two capacity units while the other firm follows with an investment of one unit followed by another investment, the sequence of investment triggers will be described by $y_{00} < y_{02} < y_{12}$; here y_{02} indicates that the firm that holds zero capacity makes an investment while the firm that holds two units is passive at that time; this leads to a capacity pair of $(1,2)$ and the notation y_{12} indicates that the same small firm acts as a leader at the third investment event of the sequence which leads to a total industry capacity of four units. A value of Y_t at which both firms make an investment will be denoted y^s_{ij}.

We will often focus on subgames, e.g. studying the next investment decision of firms that already hold some capacity units. In such cases we are interested in the value of Y_t that will trigger the next investment. Consequently, when evaluating firm values, we do not consider situations where Y_t is higher than the relevant trigger value while the corresponding investment has not been made. In other words, if, at date t, the current capacity pair is (i,j), then, depending on which firm(s) invest(s) next, $Y_t \leq y_{ij}$, or $Y_t \leq y_{ji}$, or $Y_t \leq y^s_{ij}$, or the game is over.

The next lemma gives the value of firms that hold options to invest in new capacity according to their exercise strategies. These options could be evaluated by contingent claims analysis; however the required spanning assumption would probably be violated in this imperfect competition setup. Thus we use stochastic dynamic programming.[3]

Suppose that $Y_t = y$. Let $L^\nu(i,j,y)$ $(F^\nu(i,j,y))$ denote the current value when $Y_t = y$ of the firm of capacity i if it acts as leader (follower) with regard to the next investment of ν units, while its opponent currently holds a capacity of j; similarly let $S^{00}(i,j,y)$ denote the current value when $Y_t = y$ of the firm of capacity i if no new investment by either firm occurs in the future and let $S^{\nu\nu'}(i,j,y)$ denote the current value of the firm of capacity i if the next event, occurring at $Y_\tau = y^s_{ij}$, consists of simultaneous investments of $\nu \in [1, m-i]$ units (respectively $\nu' \in [1, m-j]$ units) by the firm that holds i units (respectively the firm that holds j units). The following lemma gives the analytical expressions of the L^ν, F^ν, and $S^{\nu\nu'}$ functions.

[3]See Dixit and Pindyck (1994, Chapter 4) for a discussion of the two methods.

Lemma 1 *Let $Y_t = y$; then:*

$$\begin{aligned} L^{\nu}(i,j,y) &= \frac{\pi_{ij}}{r-\alpha}y + \left(\frac{y}{y_{ij}}\right)^{\beta}\left(\frac{\pi_{kj}-\pi_{ij}}{r-\alpha}y_{ij} - \nu I\right) + C(k,j,y)\,, \text{ for } y < y_{ij}, \\ &= \frac{\pi_{kj}}{r-\alpha}y - \nu I + C(k,j,y)\,, \text{ for } y = y_{ij}, \end{aligned}$$

where $\beta = \frac{1}{2} - \alpha/\sigma^2 + \sqrt{(\alpha/\sigma^2 - \frac{1}{2})^2 + 2r/\sigma^2} > 1$; $k = i + \nu$ *is the new capacity of the leader after investing in ν capacity units at $Y_t = y_{ij}$, and $C(k,j,y)$ is the expected effect of subsequent investments by both firms, if any, on $L^{\nu}(i,j,y)$. Similarly,*

$$\begin{aligned} F^{\nu}(i,j,y) &= \frac{\pi_{ij}}{r-\alpha}y + \left(\frac{y}{y_{ji}}\right)^{\beta}\left(\frac{\pi_{il}-\pi_{ij}}{r-\alpha}y_{ji}\right) + C(i,l,y)\,, \text{ for } y < y_{ji}, \\ &= \frac{\pi_{il}}{r-\alpha}y + C(k,j,y)\,, \text{ for } y = y_{ji}, \end{aligned}$$

where $l = j + \nu$ is the new capacity of the leader after investing in ν capacity units at $Y_t = y_{ji}$ and $C(i,l,y)$ is the expected effect of subsequent investments by both firms, if any, on $F^{\nu}(i,j,y)$. Finally,

$$S^{00}(i,j,y) = \frac{\pi_{ij}}{r-\alpha}y$$

and

$$\begin{aligned} S^{\nu\nu'}(i,j,y) &= \frac{\pi_{ij}}{r-\alpha}y + \left(\frac{y}{y_{ji}^{s}}\right)^{\beta}\left(\frac{\pi_{kl}-\pi_{ij}}{r-\alpha}y_{ji}^{s} - \nu I\right) + C(k,l,y)\,, \text{ for } y < y_{ij}^{s}, \\ &= \frac{\pi_{kl}}{r-\alpha}y - \nu I + C(k,l,y)\,, \text{ for } y = y_{ji}^{s}, \end{aligned}$$

where $k = i + \nu$ and $l = j + \nu'$ are the new capacities of the firms after investing at $Y_t = y_{ji}^{s}$ and $C(k,l,y)$ is the expected effect of subsequent investments by both firms, if any, on $S^{\nu\nu'}(i,j,y)$.

In all the above formulas, terms such as $\left(\frac{y}{y_{ij}}\right)^{\beta}$ may be viewed as discount factors defined over the state space rather than the time space: $\left(\frac{y}{y_{ij}}\right)^{\beta}$ gives the expected discounted value when $Y_t = y$ of getting one dollar the first time Y_t reaches $y_{ij} > y$.

When y is strictly lower than the value that triggers the next investment (first line in the expressions for L^{ν}, F^{ν}, and $S^{\nu\nu'}$), there are three terms in the formula. The first term gives the value of the firm at t if no further investment ever takes place. The second term gives the correction required in order to take account of the next investment while ignoring the impact of any subsequent investment. In the case of L^{ν}, this investment occurs when Y_t reaches y_{ij} for the first time and involves, for the leader, an increase $(\pi_{kj} - \pi_{ij})\, y_{ij}$ in current profit which is capitalized as a perpetuity at the rate $r - \alpha$, and reduced by the amount of the investment expenditure νI; the net expected capitalized value is then discounted back to the current date t (i.e. to the current state y) using the coefficient $\left(\frac{y}{y_{ij}}\right)^{\beta}$. For a follower, i.e. in the case of F^{ν}, the second term also gives the correction to the first term required in order to take account of the next investment; however, the next industry investment is made by the other firm so that there is no expenditure for the follower, and the change in current profit $(\pi_{il} - \pi_{ij})\, y_{ji}$ is non positive. Finally the third term $C(\cdot)$ accounts for subsequent investments if any: just as the second term corrects the first term, $C(\cdot)$ involves a correction reflecting the fact that the profit flow in the second term is not a perpetuity. If necessary, $C(m, l, y)$ can be made explicit by working backward from the horizon as with continuation functions in dynamic programming.[4]

[4] Although $C(\cdot)$ is not given an explicit form in Lemma 1, the formula are easily extended to account for further investments beyond the next one. Take the firm whose value is $F^{\nu}(i, j, y)$ as example; although a follower regarding the next investment which takes place at y_{ji}, that firm may lead or follow in subsequent ones. Suppose that, in the period immediately following the next investment it makes a one unit investment. The value of Y_t that triggers this new investment is denoted y_{il} and the change in current profit is $(\pi_{ml} - \pi_{il})\, y_{il}$ whose capitalized impact, net of the expenditure can be explicitely introduced in the expression for $F^{\nu}(i, j, y)$ (third term on the right). This new term was previously part of the unknown function $C(i, l, y)$ so that a new $C(\cdot)$ function appears in the formula: with $y < y_{ji} < y_{il}$,

$$F^{\nu}(i, j, y) = \frac{\pi_{ij}}{r - \alpha} y + \left(\frac{y}{y_{ji}}\right)^{\beta} \left(\frac{\pi_{il} - \pi_{ij}}{r - \alpha} y_{ji}\right) + \left(\frac{y}{y_{il}}\right)^{\beta} \left(\frac{\pi_{ml} - \pi_{il}}{r - \alpha} y_{il} - I\right) + C(m, l, y)$$

Similarly, for $y = y_{ji} < y_{il}$,

$$F^{\nu}(i, j, y) = \frac{\pi_{il}}{r - \alpha} y + \left(\frac{y}{y_{il}}\right)^{\beta} \left(\frac{\pi_{ml} - \pi_{il}}{r - \alpha} y_{il} - I\right) + C(m, l, y)$$

When y is such that the investment under consideration occurs immediately (second alternative in the expressions for L^{ν}, F^{ν}, and $S^{\nu\nu'}$), then the first two terms just discussed reduce to a single one which expresses the expected value of the perpetuity that would start with the current investment as if it was the last one; for a leader, or in case of simultaneous investment, the investment expenditure must be subtracted from that amount. In the sequel, we will omit the ν exponent(s) if the investment under consideration is a one unit investment.

3.2. Markov strategies and Markov perfect equilibrium

The Markov Perfect Equilibrium (MPE) is the relevant equilibrium concept for our long run dynamic investment game. In such an equilibrium, the investment decisions at any time depend only on the current state of demand represented by Y_t and the firms' current capacities. We first extend Fudenberg and Tirole's (1985) concept of preemption strategies to our stochastic framework with multiple investment and we define the Markov perfect equilibria of the duopoly.

Definition 1 *A Markov strategy for firm f is defined as a collection of functions $s^f = \left\{ s(i,j,\cdot) \in \Delta(\{0,...,m-i\})^{[0,\infty)} \mid (i,j) \in \{0,...,m\}^2 \right\}$ from $[0,\infty)$ to the $(m-i+1)$-simplex $\Delta(\{0,...,m-i\})$. The ν-th element of $s(i,j,\cdot)$ is the strategy function $s_{\nu}(i,j,y)$ giving the intensity with which a firm invests into ν additional capacity units when it currently holds i units, its competitor holds j units, and the state of demand is $Y_t = y$.*

Obviously no firm will acquire more than m units, the minimum capacity at which a monopoly is unconstrained, so that $\nu \leq m-i$. The details of the construction are gathered in the Appendix.

Definition 2 *A Markov perfect equilibrium (MPE) of the duopoly game is a subgame perfect equilibrium in Markov strategies.*

3.3. Game end

The following proposition gives partial answers to the questions raised about a firm's ability to maintain a competitive advantage. We define the game to be over if and only if, in equilibrium, it is certain that no firm will ever make any new investment in the future.

Proposition 1 *Suppose that the investment game imposes no restrictions on capacity. Then, in order for the game to be over, it is necessary that either* A *or* B *holds:*

(A) *Both firms f and g hold a strictly positive capacity and neither capacity constraint is binding in the short-run Cournot game, that is* $K^i \geq k^c = \min\{k \in \mathbb{N} \,|\, k \geq x^c\}$, $i = \{f, g\}$.

(B) *Both firms hold a strictly positive capacity, and both capacity constraints are binding in the short-run game and would remain binding in case of a unit investment by any one firm.*

Furthermore Condition A *is sufficient for the game to be over.*

The proposition indicates that no firm can keep its opponent out of the market in the long run, and that a firm cannot use excess capacity in order to maintain a dominant position in the long run. It eliminates the situation found in Gilbert and Harris (1984) where, in equilibrium, one duopolist concentrates the totality of industry capacity, while the other firm holds no capacity. Their result can be traced to a technical assumption, rightly claimed to be "trivial in that both firms will earn zero profits on new investments in a preemption equilibrium" (p. 206), that gives a first-mover advantage to one firm in order to rule out (mistaken) simultaneous investments. The strategies and equilibrium concept defined above avoid the necessity of any asymmetric treatment. In what follows firm asymmetry will only be able to take the form of differences in current capacities and may be thought of as inherited from past moves in the industry development game.

4. MPE with no existing capacity: preemption

In order to keep the analysis simple, we start with a situation where initial capacities are zero, both firms have at most one unit to invest, and $x^c > 1$ so that making the investment is attractive to both firms. Without loss of generality, we also assume that the initial demand conditions are such that no firm makes its investment immediately.

4.1. Preliminaries

Using Lemma 1, the payoff from following is $F(0,0,y) = 0 + C(0,1,y)$. Once the follower enters, she shares the market with the leader, and her instantaneous profit is $Y_t \pi_{11}$, so that $C(0,1,y) = \left(\frac{y}{y_{01}}\right)^{\beta} \left(\frac{\pi_{11}}{r-\alpha} y_{01} - I\right)$. The stopping problem faced by the follower is therefore:

$$F^*(0,0,y) = \sup_{y_{01}} \left[\left(\frac{y}{y_{01}}\right)^{\beta} \left(\frac{\pi_{11}}{r-\alpha} y_{01} - I\right) \right], \tag{4}$$

and its solution is:

$$y_{01}^* = \frac{r-\alpha}{\pi_{11}} I \frac{\beta}{\beta - 1}. \tag{5}$$

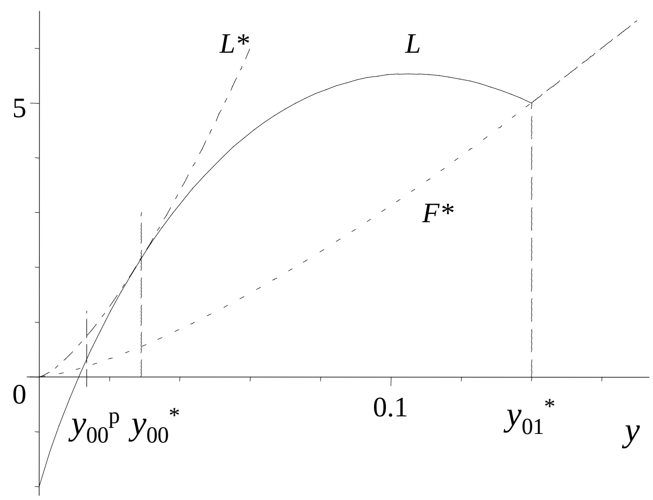
Knowing this, the value of investing as leader at $Y_t = y$, when no firm has moved yet, is:

$$L(0,0,y) = \frac{\pi_{10}}{r-\alpha} y - I + \left(\frac{y}{y_{01}^*}\right)^{\beta} \left(\frac{\pi_{11} - \pi_{10}}{r-\alpha} y_{01}^*\right), \; y = y_{00} < y_{01}.$$

Here, $\left(\frac{y}{y_{01}^*}\right)^{\beta} \left(\frac{\pi_{11}-\pi_{10}}{r-\alpha} y_{01}^*\right)$ is the explicit form taken by $C(1,0,y)$ in the expression for $L^1(0,0,y)$ given in Lemma 1. $C(1,0,y)$ is the expected effect on $L(0,0,y)$ of the investment made by the follower when Y_t reaches y_{01}^*. It has two components: $\left(\frac{y}{y_{01}^*}\right)^{\beta} \frac{\pi_{11}}{r-\alpha} y_{01}^*$ is the expected value of the cumulative cash flow that starts at y_{01}^* for the leader, when its instantaneous cash flow changes from $\pi_{10} y_{01}^*$ to $\pi_{11} y_{01}^*$; and $-\left(\frac{y}{y_{01}^*}\right)^{\beta} \frac{\pi_{10}}{r-\alpha} y_{01}^*$ is the correction to the first term in $L(0,0,y)$ accounting for the fact that the cash flow of

$\pi_{10} Y_t$ ends at y_{01}^* rather than being a stochastic perpetuity. Figure 1 illustrates the functions $L(0,0,y)$ and $F^*(0,0,y)$ if the leader invests at the current level of Y_t.

Figure 1: Firm values under alternative strategies



Similarly, if the investment is to take place in the future, i.e. for $y < y_{00}$,

$$L(0,0,y) = \left(\frac{y}{y_{00}}\right)^{\beta} \left(\frac{\pi_{10}}{r-\alpha} y_{00} - I\right) + \left(\frac{y}{y_{01}^*}\right)^{\beta} \left(\frac{\pi_{11} - \pi_{10}}{r-\alpha} y_{01}^*\right), \; y < y_{00}. \tag{6}$$

Its maximum $L^*(0,0,y)$ with respect to y_{00}, also illustrated in Figure 1, is reached at:

$$y_{00}^* = \frac{r-\alpha}{\pi_{10}} I \frac{\beta}{\beta - 1}. \tag{7}$$

4.2. Preemption and rent equalization

Consider an equilibrium random path of the game where firm f enters first and firm g second. As just shown, firm g's entry occurs at the first time where Y_t reaches y_{01}^*. However, if firm f was to achieve a higher value than its rival by investing first, firm g would attempt to preempt by investing earlier. Consequently, firm f's entry threshold should be such as to achieve the highest possible expected value that does not exceed firm

g's value as a follower, and reciprocally. Therefore, intuition suggests that in an MPE, firms invest earlier than the value of Y_t that maximizes (6) to avoid preemption, and that rents are equalized in equilibrium. The following lemma establishes the existence of a rent equalizing threshold in the relevant interval.

Lemma 2 *There is exactly one value* $y_{00}^p \in (0, y_{00}^*)$ *such that* $L(0,0,y_{00}^p) = F^*(0,1,y_{00}^p)$.

The reasoning is analogous to that in Fudenberg and Tirole (1985). From an ex ante point of view, the maximum that each firm can achieve by investing first is $L^*(0,0,y)$. This is strictly higher than $F^*(0,0,y)$. But if a firm anticipates that its rival will make its unit investment at y_{00}^*, it is better-off avoiding being preempted by investing at $y_{00}^* - dy$. Reasoning backwards, at any y between y_{00}^p and y_{00}^*, each firm wants to preempt to avoid being preempted later on. Now, if Y_t is below y_{00}^p, it is a dominant strategy not to invest because the follower's expected payoff exceeds the leader's payoff and the joint-adoption payoff. Conversely, if Y_t is above y_{01}^*, it becomes a dominant strategy for the firm with zero capacity to invest. Hence, our candidate MPE exhibits diffusion, with investment dates $\tau(y_{00}^p)$ and $\tau(y_{01}^*)$. Since $y_{00}^p < y_{00}^*$, the first entry occurs earlier than if the leader was protected against preemption. By Lemma 2 rents are equalized and are strictly lower than the rent of a protected leader that could enter at y_{00}^* without being preempted. That is the usual rent dissipation result.

The following proposition, which characterizes the equilibrium outcomes, is a transposition of Fudenberg & Tirole (1985, Proposition 2(A)) into a stochastic context:

Proposition 2 *(Preemption equilibrium) Under Assumption 1*

1. *There exists only one MPE outcomes: one firm invests when* Y_t *reaches* y_{00}^p *and the other firm invests when* Y_t *reaches* y_{01}^*.

2. *Rents to both firms are equalized to the value of the follower given by* (4).

3. *The first industry capacity unit is introduced earlier than if the leader did not face a threat of preemption:* $y_{00}^p < y_{00}^*$.

Hence, in the MPE, one of the firms enters with certainty when Y_t reaches y_{00}^p and the other firm enters when Y_t reaches y_{01}^*. There is no possibility of entry mistake at y_{00}^p, though, because, in the equilibrium strategies, the 'intensity of investment' in state y_{00}^p is null: $s_1^f(0,0,y_{00}^p) = s_1^g(0,0,y_{00}^p) = 0$. (see Appendix A, Case 2)

The preemption MPE is characterized by intense competition. Let us define welfare as the discounted sum of the maximum that consumers are willing to pay for the production of each capacity slice, minus capacity expenditures. As we state in Lemma 3, this surplus can be decomposed into the sum of the independent contributions of all successive units so that each investment threshold may be optimized separately in the process of welfare maximization.

Lemma 3 *The cumulative expected surplus when demand state is $Y_t = y$ is:*

$$\Phi(y) = \sum_{i=0}^{\bar{\imath}-2} \left(\frac{y}{y_i}\right)^{\beta} \left(\frac{D^{-1}(i+1)}{r-\alpha} y_i - I\right),$$

where i is the number of capacity units present in the industry before y_i is reached for the first time and $\bar{\imath}$ is the number of units that saturates demand.

Consider the first unit; the threshold is:

$$\begin{aligned} y_0^* &= \arg\max_{y_0} \left(\frac{y}{y_0}\right)^{\beta} \left(\frac{D^{-1}(1)}{r-\alpha} y_0 - I\right) \\ &= \frac{r-\alpha}{\pi_{10}} I \frac{\beta}{\beta-1}. \end{aligned}$$

This socially optimal investment threshold coincides with y_{00}^* the threshold that a leader would choose if it was not threatened by preemption.[5] Since $y_{00}^p < y_{00}^*$, we state:

[5]The maximization problem that defines y_0^* is precisely the problem a myopic firm would solve if it did not consider the impact of future investments (by other firms and by itself) on its own value. Leahy (1994) has shown that this behavior is optimal and coincides with competitive industry equilibrium when firms are small relative to the market.

Proposition 3 *In a preemption equilibrium with no existing capacity, the first industry unit is introduced earlier than is Pareto optimal. If any one firm was protected against preemption, it would introduce the first capacity unit at the Pareto optimal time.*

In the initial industry development phase, competition is more intense under duopoly than socially desirable. The leader must waste resources during that phase in order to compensate for future earnings in such a way that, in total, his net rent does not exceed that of the follower.[6] Thus in the preemption equilibrium the industry produces in a phase of market development where output should optimally be zero. In that sense the timing distortion is also a distortion in production. Excessive production is not a permanent characteristic of the preemption equilibrium either. For example, under the assumptions of Proposition 3 it is easily shown that the second unit of industry capacity is introduced at the socially optimal date, although this result is not robust to the alternative setups examined further below.

Empirically, excessive production or premature investment would be very difficult to identify. However, the real options approach to project evaluation indicates that a non strategic investor would find it uneconomical to invest at any $Y_t \leq y_0^*$ because that would mean insufficient protection against bad realizations of Y. The net present value of the project, as corrected to include the value of the flexibility lost at the time of investment, is negative. In that sense, the model predicts that the leader is taking too much risk, with the empirical implication that lower than normal returns, perhaps also a higher than normal incidence of bankruptcies, should be observed during the phase immediately following the first industry investment. This happens although it is known that the market will develop over the long run. Intense competition destroys value in the early phase of market development: the preemption motive overwhelms the option value.

[6] The analysis is readily adapted if one uses a continuous concept of consumer surplus rather than the stepwise function adopted here. In that case consumer surplus triangles must be added to each capacity slice, so that the socially optimal triggers are slightly lower.

5. Existing capacity: collusion and preemption in the symmetric case

Let us now investigate the role of existing capacity. Suppose that each firm holds one capacity unit and, as before, each may acquire at most one more unit. To focus on the question of interest, we assume that the unconstrained Cournot equilibrium is such that $x^c > 1$. Consequently, for high enough values of y, it is profitable for both firms to invest if the other one never does. Furthermore, even if one firm has taken the lead, the other one remains capacity constrained so that it finds it profitable, for high enough values of y, to invest as a follower.[7] Thus the following assumption:

Assumption 2 $\pi_{21} > \pi_{11}$ *and* $\pi_{22} > \pi_{12}$;

When considering a new investment, the firms will now take account of the consequences on the profits they make from their existing capacity unit. As a result of the cannibalism effect now present in the game, we will show that tacit-collusion equilibria may exist besides the preemption equilibrium, provided that late joint investment, or no investment at all, dominates leading over the relevant market development range.

Consider the preemption equilibrium. The relevant value functions are, using Lemma 1:

$$F^*(1,1,y) = \sup_{y_{12}} \left[\frac{\pi_{12}}{r-\alpha} y + \left(\frac{y}{y_{12}} \right)^{\beta} \left(\frac{\pi_{22} - \pi_{12}}{r-\alpha} y_{12} - I \right) \right], \; y_{11} \leq y < y_{12}, \tag{8}$$

where the maximum is reached at:

$$y_{12}^* = \frac{r-\alpha}{\pi_{22} - \pi_{12}} I \frac{\beta}{\beta - 1}. \tag{9}$$

Thus it is a dominant strategy for the follower to invest at y_{12}^* if the leader has already entered; taking that into account, the value of leading at the current level of y is:

$$L(1,1,y) = \frac{\pi_{21}}{r-\alpha} y - I + \left(\frac{y}{y_{12}^*} \right)^{\beta} \left(\frac{\pi_{22} - \pi_{21}}{r-\alpha} y_{12}^* \right), \; y = y_{11} < y_{12}^*. \tag{10}$$

[7] This is not an additional assumption but a consequence of $x^c > 1$.

Absent any threat of preemption the optimum investment threshold, established as (7), is given by:

$$y_{11}^{*} = \frac{r-\alpha}{\pi_{21}-\pi_{11}} I \frac{\beta}{\beta-1}.$$

In the preemption equilibrium, one of the firms takes the lead in buying its second unit, while the other one follows later on. The leader cannot wait until Y_t reaches y_{11}^{*} for fear of being preempted. As before the leader invests at some lower level y_{11}^{p} at which rents are equalized; any potential leader's advantage is dissipated by early investment.

The following lemma, analogous to Lemma 3, establishes the existence of a threshold which equalizes the rent of the leader and the rent of the follower in the game starting when both firms hold one capacity unit. As shown in Proposition 4, this implies that the preemption equilibrium always exists and is unique.

Lemma 4 *There is exactly one value $y_{11}^{p} \in (0, y_{11}^{*})$ such that $L(1,1,y_{11}^{p}) = F(1,1,y_{11}^{p})$.*

Consider now strategies consisting in abstaining from investing forever, or in simultaneous entry. These are the only possible alternative strategies since the only equilibrium compatible with investments at different dates is the preemption equilibrium. We call these strategies "tacit collusion" as they imply some implicit coordination to increase rents over their preemption level. The relevant payoff functions are, in case of tacit collusion by inaction:

$$S^{00}(1,1,y) = \frac{\pi_{11}}{r-\alpha} y,$$

and, in case of tacit collusion with simultaneous investment:

$$S(1,1,y) = \frac{\pi_{11}}{r-\alpha} y + \left(\frac{y}{y_{11}^{s}}\right)^{\beta} \left(\frac{\pi_{22}-\pi_{11}}{r-\alpha} y_{11}^{s} - I\right).$$

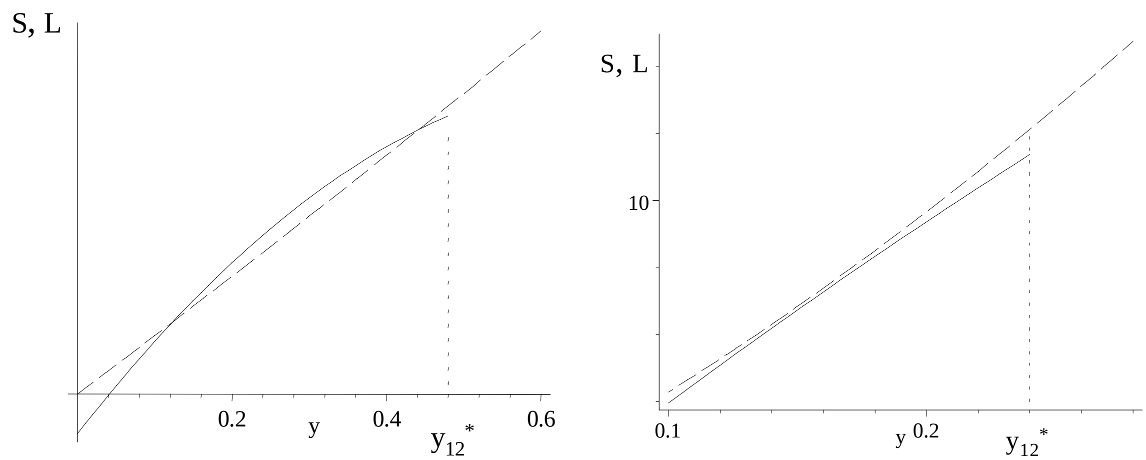
If $\pi_{22} > \pi_{11}$, $S(1,1,y)$ has a maximum with respect to y_{11}^{s}, denoted $S^{*}(1,1,y)$, when:

$$y_{11}^{s*} = \frac{r-\alpha}{\pi_{22}-\pi_{11}} I \frac{\beta}{\beta-1} > y_{12}^{*}.$$

If $\pi_{22} \leq \pi_{11}$, $S(1,1,y)$ is maximized by letting $y_{11}^s \to \infty$ in which case $S^*(1,1,y) = S^{00}(1,1,y)$. Thus we may treat tacit collusion by inaction as a special case of tacit collusion with investment, and we have $S^*(1,1,y) \geq S^{00}(1,1,y)$ with a strict inequality if $\pi_{22} > \pi_{11}$ and with equality otherwise.[8]

In case of tacit collusion, the best the firms can do is to invest simultaneously at the first time Y_t reaches y_{11}^{s*}, or to abstain forever. Since they have identical capacities before and after the joint investment, they get the same payoff: thus y_{11}^{s*} is the joint investment threshold that maximizes joint profits. This cannot be an equilibrium in the configuration of Figure 2a because $L(1,1,y) > S^*(1,1,y)$ over some interval below y_{12}^* so that both firms would rather be leaders than wait. However, in the case of Figure 2b, there is no value of $y < y_{12}^*$ at which $L(1,1,y) \geq S^*(1,1,y)$. In that case, tacit-collusion equilibria exist. In fact any y_{11}^s such that $y_{12}^p < y_{11}^s < y_{11}^{s*}$ and $L(1,1,y) \leq S(1,1,y)$ $\forall y < y_{12}^*$ may sustain a collusive MPE, the best one from the industry point of view being defined by simultaneous entry at y_{11}^{s*}. In a tacit-collusion equilibrium, rents are equalized and both firms earn higher rents than in the preemption equilibrium.

Figure 2: Leading (solid) versus Tacit collusion (dash)



a. Tacit collusion equilibrium does not exist b. Tacit collusion equilibrium exists

[8]For example, if the inverse demand is $P = \left(1 - \frac{X_1+X_2}{a}\right) Y_t$, $\pi_{22} \leq (>)\ \pi_{11}$ according to whether $a \leq (>)\ 6$ and Assumption 2 is satisfied if $a \geq 3$. Consequently, tacit collusion by inaction is the best form of tacit collusion when a lies between 3 and 6.

Proposition 4 *(Equal capacities) Under Assumptions 1 and 2, there are two possible types of MPE:*

1. *The preemption equilibrium: one firm invests when Y_t reaches y_{11}^p and the other firm invests when Y_t reaches y_{12}^*; rents to both firms are equalized to the value of the follower given by* (8)*; this equilibrium always exists;*
2. *Tacit-collusion equilibria (simultaneous investment or no investment):*
   (a) *tacit-collusion equilibria exist if and only if $L(1,1,y) \leq S^*(1,1,y) \;\; \forall y \in (0, y_{12}^*]$; if $\pi_{22} > \pi_{11}$, the best tacit-collusion equilibrium has both firms investing when Y_t reaches y_{11}^{s*}; otherwise the best tacit-collusion equilibrium is such that neither firm ever invests; rents are equalized and exceed $F^*(1,1,y)$;*
   (b) *if the set of tacit-collusion equilibria is not empty, it includes the joint-profits maximizing investment rule, with payoff $S^*(1,1,y)$ for each firm.*

This proposition highlights the role of existing capacity. When a firm does not hold any capacity it does not lose if it does not tacitly collude, so that non cooperative collusion cannot be enforced.[9] As was also the case in Fudenberg and Tirole, if tacit-collusion equilibria exist, both firms get higher payoffs in any such equilibrium than in the preemption equilibrium; if they could coordinate they would choose the joint-profits maximizing equilibrium.[10]

We further show how the volatility and the speed of market development affect the existence of collusive equilibria. In a nutshell, tacit collusion among duopolists of equal size is more likely in fast-growing and/or high-volatility markets:

[9] That argument applies to any initial situation with one firm holding no capacity and one firm holding positive capacity: there exists a level of Y at which the firm with zero capacity can make a positive expected profit by investing, which is better than zero, its tacit-collusion payoff.

[10] The joint-profits maximizing equilibrium is subject to the simultaneous investment condition. Absent that constraint, as under monopoly, joint profits maximization would involve sequential investments. This cannot be an equilibrium under duopoly to the extent that such Pareto optimal sequence would yield higher payoffs to the first investor and lead to preemption.

Proposition 5 *(Tacit collusion with equal capacities) Under Assumptions 1 and 2:*

1. *There exists a level of volatility above which tacit-collusion equilibria exist.*
2. *There is an admissible discount rate below which tacit-collusion equilibria exist.*
3. *There is an admissible expected market growth rate above which tacit-collusion equilibria exist.*

In a non strategic real option context, increased volatility increases the value of flexibility and raises the investment threshold: in order to be undertaken, the irreversible project must generate a higher expected present value because the value of the option to remain flexible is higher. This effect is also present here; β, as defined in Lemma 1, is a decreasing function of σ, so that investment thresholds y_{00}^*, y_{01}^* etc. are rising in σ. However there is a further effect of volatility, that reinforces the first one. An increase in volatility may give existence to collusive equilibria. Such equilibria involve both higher firm values and higher investment thresholds than the preemption equilibrium.

6. Different capacities: catching up, reinforcement, or collusion?

While we have shown that existing capacity is a necessary condition for collusion, it may also play a role as a barrier to entry and thus be used as a way to acquire and maintain a dominant position or a first-mover advantage. In this Section, it is assumed that both firms may invest at most one unit, but differ in their initial sizes. We show that, in the preemptive equilibrium of a game that starts with different capacity levels, it is the smaller firm that invests first. We also show that, if tacit collusion is possible in equilibrium, it is not as attractive as with firms of equal size in the sense that joint-profits maximizing investment is not compatible with equilibrium.

Suppose that one firm holds two capacity units while the other holds one unit. To focus on the interesting case, let us assume also that the portion of market demand that remains unfulfilled is such that, for high enough values of y, it is profitable for both firms to invest if the other never does.

Assumption 3 $\pi_{31} > \pi_{21}$, $\pi_{22} > \pi_{12}$.

An implication of that assumption is that the smaller firm finds it profitable to invest as a follower in case the other firm has taken the lead:

Lemma 5 $\pi_{23} > \pi_{13}$.

The optimized values for the bigger firm and for the smaller firm to act as a follower are respectively $F^*(2,1,y)$ and $F^*(1,2,y)$. Precisely, for the bigger firm,[11] if it allows its opponent to be the first to increase its capacity,

$$F^*(2,1,y) = \sup_{y_{22}} \left[\frac{\pi_{22}}{r-\alpha} y + \left(\frac{y}{y_{22}} \right)^\beta \left(\frac{\pi_{32} - \pi_{22}}{r-\alpha} y_{22} - I \right) \right], \; y_{12} < y \leq y_{22}.$$

For the smaller firm,

$$F^*(1,2,y) = \sup_{y_{13}} \left[\frac{\pi_{13}}{r-\alpha} y + \left(\frac{y}{y_{13}} \right)^\beta \left(\frac{\pi_{23} - \pi_{13}}{r-\alpha} y_{13} - I \right) \right], \; y_{21} < y \leq y_{13}.$$

Let y_{22}^* and y_{13}^* be the respective investment triggers for the bigger and the smaller firms acting as followers.[12] If the bigger firm becomes leader at $Y_t = y_{21}$,

$$L(2,1,y) = \begin{cases} \frac{\pi_{21}}{r-\alpha} y + \left(\frac{y}{y_{21}} \right)^\beta \left(\frac{\pi_{31}-\pi_{21}}{r-\alpha} y_{21} - I \right) + \left(\frac{y}{y_{13}^*} \right)^\beta \left(\frac{\pi_{32}-\pi_{31}}{r-\alpha} y_{13}^* \right) & \text{if } y < y_{21}, \\ \frac{\pi_{31}}{r-\alpha} y - I + \left(\frac{y}{y_{13}^*} \right)^\beta \left(\frac{\pi_{32}-\pi_{31}}{r-\alpha} y_{13}^* \right) & \text{if } y = y_{21} < y_{13}^*. \end{cases}$$

[11] Once the smaller firm has invested, the instantaneous profit of the bigger firm is $\pi_{22} Y_t$; then it chooses its own investment trigger y_{22}^* without any constraint.

[12] $y_{13}^* = \frac{1}{\pi_{23}-\pi_{13}} (r-\alpha) I \frac{\beta}{\beta-1}$ and exists by Lemma 3. $y_{22}^* = \frac{1}{\pi_{32}-\pi_{22}} (r-\alpha) I \frac{\beta}{\beta-1}$ if $\pi_{32} - \pi_{22} > 0$ and may be taken to be infinite if $\pi_{32} - \pi_{22} = 0$; in the latter case, $F^*(2,1,y) = \frac{\pi_{22}}{r-\alpha} y$.

If the smaller firm becomes leader at $Y_t = y_{12}$,

$$L(1,2,y) = \begin{cases} \frac{\pi_{12}}{r-\alpha}y + \left(\frac{y}{y_{12}}\right)^{\beta}\left(\frac{\pi_{22}-\pi_{12}}{r-\alpha}y_{12} - I\right) + \left(\frac{y}{y_{22}^*}\right)^{\beta}\left(\frac{\pi_{23}-\pi_{22}}{r-\alpha}y_{22}^*\right) \text{ if } y < y_{12}, \\ \frac{\pi_{22}}{r-\alpha}y - I + \left(\frac{y}{y_{22}^*}\right)^{\beta}\left(\frac{\pi_{23}-\pi_{22}}{r-\alpha}y_{22}^*\right) \text{ if } y = y_{12} < y_{22}^*. \end{cases}$$

If the small firm did not face any threat of preemption, it could maximize $L(1,2,y)$ by choosing:

$$y_{12}^* = \frac{r-\alpha}{\pi_{22}-\pi_{12}} I \frac{\beta}{\beta-1}. \tag{11}$$

Two alternative candidate preemption equilibria may be considered: one where the bigger firm acts as leader and the smaller firm acts as follower; another one where the roles are reversed. Unlike the case with symmetric initial capacities, it turns out that an equilibrium exists only in the second instance. In order to prove that result, we need the following lemma.

Lemma 6 *If $L(2,1,y) > F^*(2,1,y)$ for some $y < y_{13}^*$, then there is exactly one value $y_{12}^p \in (0, y_{13}^*)$ such that $L(2,1,y_{12}^p) = F^*(2,1,y_{12}^p)$ and $L(2,1,y) < F^*(2,1,y)$ for $y < y_{12}^p$.*

The lemma indicates that, by becoming leader at $y = y_{12}^p$, earlier than the stand-alone situation corresponding to y_{12}^*, the smaller firm can preempt while leaving the bigger firm indifferent between preempting or following. Furthermore we show in the proof of the next proposition that, at $y = y_{12}^p$, the smaller firm strictly prefers becoming leader rather than following, and that, at any other relevant level of y, the gain for the bigger firm to become leader is smaller than the gain for the smaller firm to lead. These results imply that the sole preemption equilibrium is one where the smaller firm catches up. Trivially, if the bigger firm cannot gain by investing, then the smaller firm need not worry about preemption and invests at its stand-alone date. This case is also covered in Proposition 6 below, when $\min\{y_{12}^p, y_{12}^*\} = y_{12}^*$.

Proposition 6 *(Preemption with different capacities) Under Assumptions 1 and 3:*

1. *There exists a unique preemption equilibrium, where the smaller firm invests when Y_t first reaches* $\min\{y_{12}^p, y_{12}^*\}$*, and the bigger firm acts as follower;*

2. *In the preemption equilibrium, the smaller firm enjoys a strictly positive rent from leading:* $L(1,2,y_{12}^p) - F(1,2,y_{12}^p) > 0$*; while the bigger firm is either indifferent between leading and following, or prefers following:* $L(2,1,y_{12}^p) \leq F^*(2,1,y_{12}^p)$*.*

In the preemption equilibrium the laggard not only catches up but also enjoys an advantage in terms of rent. The reason is not because the laggard is in a better position to avoid immediate cannibalism. Although the drop in revenues from existing capacity is indeed smaller for the smaller firm than for the bigger firm when industry output increases, the same drop in price occurs, in a preemption context, whether it is caused by the investment of one or the other firm. Thus the source of the first-mover advantage must be found in future decisions rather than current effects. Once one firm has made its investment, the other firm plans its own investment in a non strategic setup, at its stand-alone date. It is solely responsible for the effect of the drop in price on its current revenues. Having less to loose from the cannibalism effect, a smaller firm invests earlier in the future as follower than a bigger firm would. This reduces the advantage enjoyed by its opponent from taking the lead.

Possible sources of first-mover advantage or rationale for dominant position have been considered repeatedly in the literature. In Stiglitz and Dasgupta (1988), the fact that contesting a dominant firm is costly is enough to secure the latter's dominant position. Although investment is costly in our model, this argument does not apply here because the market develops, so that competition is not over current sales but over the next capacity investment. No firm enjoys any cost advantage over that investment.

In dynamic situations - patents races or investment games - the issue has often been whether an exogenous advantage in terms of timing could generate rents. In Gilbert and Harris (1984) this does not prevent rent dissipation. In Mills (1988), the exogenous ability to move first can be used to make a costly preliminary investment which works as a threat that keeps the rival at bay and thus generates rents for the first mover.

Similarly, in patent race games, Fudenberg *et al.* (1983) and Harris and Vickers (1985) have established that when a firm exogenously gets an arbitrarily small head start, there is a unique perfect-Nash equilibrium in which the firm with the head start surely wins. In the present paper, exogenous differences may only result from past capacity investments. It is remarkable that being big is not like enjoying a head start in a race; quite the contrary, being big makes the threat of early subsequent investment less credible.

The preemption equilibrium of Proposition 6 is unique in the class of equilibria involving investment by both firms at different dates. As with equal capacities, there may exist another class of equilibria, involving simultaneous investment or inaction. As shown in the next proposition, if volatility is high enough or if market growth is fast enough, such tacit-collusion equilibria exist. However tacit collusion loses some of its attractiveness when firms hold different capacities in the sense that joint-profit maximization cannot be achieved as a tacit-collusion equilibrium. Being different firms would prefer different thresholds for simultaneous investment, and the smaller firm would defect at the threshold that maximizes joint profits.

Proposition 7 *(Tacit collusion with different positive capacities) Under Assumptions 1 and 3:*

1. *No tacit-collusion equilibrium exists if* $\pi_{32} - \pi_{22} \leq 0$*;*

2. *If* $\pi_{32} - \pi_{22} > 0$*: there exists a level of volatility above which tacit-collusion equilibria exist; there is a discount rate below which tacit-collusion equilibria exist; there is a rate of expected market growth above which tacit-collusion equilibria exist;*

3. *Joint-profits maximization is not compatible with tacit-collusion equilibrium.*

If one firm holds no capacity while the other one does, a simple adaptation of the proof shows that the firm without capacity is better off leading than following. Furthermore, since it does not loose any revenues from existing capacity when it introduces new capacity, that firm is also better off leading (alone) rather than investing later simultaneously with its opponent. Consequently, irrespective of volatility, discount rate or market growth, the tacit-collusion equilibrium does not exist if one firm holds no capacity. In the early phase of development of an industry, when only one firm is active, competition is fierce in the sense that the sole equilibrium is the preemption equilibrium.

7. Conclusion

We have presented a methodology that allows the study of real-options investment decisions in a strategic duopoly setup, with the formulation of appropriate payoff functions and the generalization to a stochastic context of Fudenberg and Tirole (1985)'s formalism for defining strategies in a continuous-time environment.

Applying this methodology to specific special cases has allowed us to identify some properties and stylized characteristics of industries that develop under duopoly, when the investment required are indivisible, irreversible, and big relative to the market. While many other considerations affect industry development, we think that the magnitude and irreversibility of outlays relative to market size are important considerations not only in young sectors, especially those involving scale economies, but also in more conventional and older ones such as the aircraft industry. The speed of market development and the uncertainty regarding its future evolution are also important considerations that the real options approach is well equipped to handle.

In our model, the indivisible capacity unit is costly and never becomes small relative to the market despite unbounded market development. Nonetheless we show that one firm cannot durably keep its opponent at bay by holding as many capacity units as the market can bear.

We have found that the early phase of such an industry is characterized by strong competition in the sense that one firm preempts the other. This competition causes the first industry investment to occur earlier than would be socially optimal, a distortion which implies riskier entry, lower expected returns, and more bankruptcies. This waste of resources is inevitable and allows the equalization of the rents of the leader and the follower. It occurs irrespective of the volatility or the speed of market development.

At later stages of development, when both firms hold capacity, competition may be weaker in the sense that tacit-collusion equilibria may exist. Tacit collusion to restrict production takes the form of postponed simultaneous investment by both firms. In fact tacit-collusion equilibria are sure to exist in high volatility markets or fast growth markets. Here the conventional real options result that high volatility postpones investments is reinforced by the fact that higher volatility may allow a switch from the preemption equilibrium, which always exist, to a tacit-collusion equilibrium involving later investment and higher profits.

When it exists at all, the possibility of collusion is more attractive to firms of equal size than to unequal ones. This is because a tacit-collusion equilibrium requires simultaneous investment by both firms. When firms are of equal size, this is compatible with joint profit maximization; when firms differ in size the joint-profit joint-investment threshold is beyond the level that maximizes the expected profits of the smaller firm: the latter would defect at that level of market development. This suggests that tacit collusion is less efficient as a way to raise profits the more the firms differ in size. If other forms of collusion, such as acquisitions or mergers, are possible, one would expect them to become relatively more attractive the more unequal the firm sizes.

Thus competition definitely works, but collusion is possible, and appearances may be deceiving. The stylized properties that we have outlined suggest that competition is more likely to be at work when only one firm operates and that collusion is more likely when the industry is made up of two active firms of equal size and when market develops quickly and/or with much volatility.

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Appendix A: Details on Markov strategies

Let i (resp. j) be the number of capacity units held by firm f (resp. g). A Markov strategy for firm f will be a mapping s^f that associates, to each capacity vector $(i,j) \in \{0,\dots,m\} \times \{0,\dots,m\}$ and state of demand $y \in (0,\infty)$ a probability distribution $s^f(i,j,y) \in \Delta(\{0,\dots,m-i\})$. (Since m is the maximum number of units that a monopoly would acquire, it is obvious that firms never wish to acquire more than m units in total). For each $\nu \in \{0,\dots,m-i\}$, the quantity $s^f_\nu(i,j,y)$ will be interpreted as the intensity with which firm f invests ν units when it holds i units, firm g holds j units, and $Y_t = y$. The following regularity conditions are imposed for technical convenience:

(R_1) *For each $(i,j) \in \{0,\dots,m\} \times \{0,\dots,m\}$, the mapping $s^f(i,j,\cdot)$ is piecewise continuous and admits everywhere a right limit.*

(R_2) *For each $y \in (0,\infty)$, if $s^f_0(i,j,y) = 1$, then $s^f_0(i,j,\cdot) = 1$ in a left neighborhood of y. If furthermore $y'' = \inf\{y' > y \mid s^f_0(i,j,y') \neq 1\}$, then for all $\nu \in \{1,\dots,m-i\}$, each right partial derivative $\partial^+_y s^f_\nu(i,j,y'')$ exists and at least one of them is strictly positive.*

A strategy for firm g is defined in a similar way. For each strategy profile $s = (s^f, s^g)$, let $U^f_s(i,j,y)$ denote the value of firm f when s is played and the current state of the game is (i,j,y). Since investment is irreversible and capital does not depreciate, we will be able to compute these value functions recursively from the equilibria of the static Cournot game. To do so, we distinguish three cases.

Case 1. First, if $y \in A_s(i,j) = \{y' \in [0,\infty) \mid s^f_0(i,j,y')\, s^f_0(j,i,y') \neq 1\}$, then at least one firm is active in the state (i,j,y). $U^f_s(i,j,y)$ is then defined as:

$$\frac{\sum_{(\nu^i,\nu^j)\neq(0,0)} s^f_{\nu^i}(i,j,y)\, s^g_{\nu^j}(j,i,y) \left(U^f_s(i+\nu^i, j+\nu^j, y) - \nu^i I\right)}{1 - s^f_0(i,j,y)\, s^g_0(j,i,y)}. \tag{A.1}$$

The intuition underlying (A.1) is that, if the firms' intensity of investment is not identically zero in state (i,j,y), with probability one some new investments will take place in this state. Specifically, exactly (ν^i,ν^j) additional units of capacity will be invested with probability $s^f_{\nu^i}(i,j,y)\, s^g_{\nu^j}(j,i,y)\,(1 - s^f_0(i,j,y)\, s^g_0(j,i,y))$.

Case 2. Next, if $y \in \partial A_s(i,j) \setminus A_s(i,j)$, the intensity of entry of both firms is zero. However, since y is on the boundary of $A_s(i,j)$, with probability one the first random time $\tau_s(i,j,y)$ at which the process of demand shocks reaches $A_s(i,j)$ starting from y is equal to zero. (This is a special case of the 0-1 law for diffusions, see Øksendall (1995, Corollary 9.2)). Using (R_1) and (R_2), $U^f_s(i,j,y)$ can be computed as the limit of $U^f_s(i,j,y+\varepsilon)$ when $\varepsilon \downarrow 0$, if necessary using a first-order Taylor expansion of (A.1). As in Fudenberg and Tirole (1985), the interpretation is that there is an "interval of atoms" following y in at least one firm's strategy, so with probability one some investments will occur at y.

Case 3. Last, if $y \notin A_s(i,j) \cup \partial A_s(i,j)$, both firms' intensities of investment are zero in a neighborhood of y. Firm f's continuation value is then:

$$E_y \left(\int_0^{\tau_s(i,j,y)} e^{-rt} \pi_{ij} \, Y_t dt + e^{-r\tau_s(i,j,y)} \, U_s^f(i,j,Y_{\tau_s(i,j,y)}) \right), \tag{A.2}$$

where $\tau_s(i,j,y)$ is defined as in Case 2.

Appendix B: Proofs

Proof of Lemma 1. Let $Y_t = y$. The value of a firm at date t is the expected present value of its profits over the periods between investments by either firms, minus the present value cost of the investments made by the firm. In the case of a firm of capacity i that takes the lead for the next investment, of ν capacity units, at $\tau_{ij} \geq t$,

$$L^\nu(i,j,y) = E^y \left\{ \int_t^\infty e^{-rs} \pi_{ij} Y_s ds + \int_{\tau_{ij}}^\infty e^{-rs} \left(\pi_{kj} - \pi_{ij}\right) Y_s ds - \nu e^{-r\tau_{ij}} I \right\} + C(k,j,y).$$

This formula may be decomposed as follows. At t, the current profit of the firm of capacity i is $\pi_{ij} Y_t$. If this situation was to last forever, the corresponding cumulative profits, capitalized at t, would be given by the first term in the expected value operator. However this situation lasts only until the new investment occurs at τ_{ij}; consequently, the cumulative present value at t of the profit flow $\pi_{ij} Y_s$ on $[\tau_{ij}, \infty)$ is deducted, in the second term. That term also contains the cumulative present value on $[\tau_{ij}, \infty)$ of the profit flow $\pi_{kj} Y_s$ which replaces $\pi_{ij} Y_s$ at τ_{ij}. This profit flow may again not last forever; if it is altered by some new investment after τ_{ij}, a deduction for the appropriate period is included in $C(k,j,y)$. The rest of $C(k,j,y)$ is the continuation value, and may be constructed in a similar fashion. Finally, the third term in the expected value operator accounts for the investment expenditure made by the firm at τ_{ij}.

The time homogeneity of $(Y_t)_{t \geq 0}$ and the strong Markov property for diffusions imply that, for all $y \geq 0$,

$$L^\nu(i,j,y) = E^y \left\{ \frac{\pi_{ij}}{r - \alpha} y + e^{-r\tau_{ij}} \left(\frac{\pi_{kj} - \pi_{ij}}{r - \alpha} Y_{\tau_{ij}} - \nu I \right) \right\} + C(k,j,y).$$

We are interested in stopping regions of the form $[y_{ij}, \infty)$. For any $y_{ij} > 0$, let $\tau(y_{ij}) =$

$\inf\{t > 0 \mid Y_t \geq y_{ij}\}$, so that $Y_{\tau(y_{ij})} = y_{ij}$ $P - a.s.$; then $L^\nu(i,j,y)$ may be rewritten as:

$$L^\nu(i,j,y) = E^y\left\{\frac{\pi_{ij}}{r-\alpha}y + e^{-r\tau(y_{ij})}\left(\frac{\pi_{kj}-\pi_{ij}}{r-\alpha}y_{ij} - \nu I\right) + C(k,j,y)\right\}. \tag{A.3}$$

Following Harrison (1985, chapter 3), the Laplace transform $E^y\left\{e^{-r\tau(y_{ij})}\right\}$ is, for any $y \in [0, y_{ij})$:

$$\begin{aligned} E^y\left\{e^{-r\tau(y_{ij})}\right\} &= E^y\left\{\exp\left(-r\inf\left\{t \geq \mid \left(\alpha - \frac{\sigma^2}{2}\right)t + \sigma Z_t \geq \ln\left(\frac{y_{ij}}{y}\right)\right\}\right)\right\} \\ &= \exp\left(\left[-\frac{1}{2} + \frac{\alpha}{\sigma^2} - \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}\right]\ln\left(\frac{y_{ij}}{y}\right)\right) \\ &= \left(\frac{y}{y_{ij}}\right)^\beta, \end{aligned}$$

by definition of β. Substituting into (A.3) yields the formula for $L^\nu(i,j,y)$ given in the Proposition. The other expressions are obtained in a similar way. ■

Proof of Proposition 1. *A strictly positive capacity is necessary.* Suppose one firm has zero capacity. Then its profit is zero. If it buys one unit, the lowest instantaneous profit it can make at any time after making that investment is $Y_t\pi_{1k}$, where k is the capacity at which its opponent is unconstrained in the short run in response to an output of one: this corresponds to the worst-case scenario where its opponent holds the capacity which leaves the firm the lowest instantaneous profit and the firm does not acquire any further units even if it is profitable for it to do so.

The maximized expected discounted present value from buying one capacity unit at some future time τ is, in that worst-case scenario, $V(0,k,y) = \sup_\tau E^y\left\{\int_\tau^\infty e^{-rt}Y_t\pi_{1k}dt - e^{-r\tau}I\right\}$. Using the approach of Lemma 1 to evaluate V leads to

$$V(0,k,y) = \sup_{y_{0k}}\left(\frac{y}{y_{0k}}\right)^\beta\left(\frac{\pi_{1k}}{r-\alpha}y_{0k} - I\right)$$

The value of y_{0k} that solves the maximization is

$$y_{0k}^* = \frac{I(r-\alpha)}{\pi_{1k}}\frac{\beta}{\beta-1}$$

so that $V(0,k,y) > 0$. Thus the strategy of never buying in the future is strictly dominated for the firm whose capacity is zero. In consequence both firms will hold strictly positive capacity.

Either A or B is necessary. Assume that neither A nor B holds, that is: let k and l be the respective capacities; let l be such that the corresponding firm is capacity constrained and let k be such that the firm that holds k units is not constrained if the other firm has a capacity of $l+1$ or more units. If the constrained firm increases its capacity to $l+1=n$ its current instantaneous profit increases to $\pi_{nk} > \pi_{lk}$; indeed the opponent has no alternative but to accommodate by reducing output, and will not change its capacity. The maximized gain in expected discounted present value from bringing capacity to n at some future time τ is $v(l,k,y) = \sup_{y_{lk}} \left(\frac{y}{y_{lk}}\right)^{\beta} \left(\frac{\pi_{nk}-\pi_{lk}}{r-\alpha} y_{lk} - I\right)$. This is positive, implying that a strategy of never investing in a situation where one firm is constrained, while the other is unconstrained or would become unconstrained after a unit investment by its opponent, is strictly dominated.

Condition A is sufficient. If neither capacity constraint is binding, no firm can increase profit by further investing so that the game is necessarily over.

■

Proof of Lemma 2. Straightforward computations show that $L-F$ is strictly quasi-concave in y, $L(0,0,0) - F(0,0,0) = -I$ and $L(0,0,y^*_{01}) - F(0,0,y^*_{01}) = 0$. By the intermediate value theorem, we need only to prove that $L(0,0,y^*_{00}) - F(0,0,y^*_{00})$ is positive. Taking the derivative with respect to y at $y = y^*_{00}$ yields that result. Also, since $L(0,0,y^p_{00}) - F(0,0,y^p_{00}) = 0$ and $L(0,0,y^*_{00}) - F(0,0,y^*_{00}) > 0$, $y^p_{00} < y^*_{00}$. ■

Proof of Proposition 2. #1 and #2. Let:

$$s_1^f(0,0,y) = s_1^g(0,0,y) = \begin{cases} 0, \ y \in [0, y^p_{00}) \\ \frac{L(0,0,y)-F^*(0,0,y)}{L(0,0,y)-S(0,0,y)}, \ y \in [y^p_{00}, y^*_{01}) \\ 1, \ y \in [y^*_{01}, \infty) \end{cases}.$$

where, from Lemma 1, for a simultaneous investment occurring at $y^s_{00} = y$, $S(0,0,y) = \frac{\pi_{11}}{r-\alpha}y - I$. We have shown already that, on $[0, y^p_{00})$, it is a dominant strategy not to invest, and on (y^*_{01}, ∞), it is dominant for a firm with zero capacity to invest if the other holds one unit. We now show that the above strategy combination form a Nash equilibrium in any subgame starting at $y \in [y^p_{00}, y^*_{01})$. For $y \in [y^p_{00}, y^*_{01})$, if firm f deviates by choosing $s'(0,0,y) = 0$, the other firm enters at y so that firm f's dominant strategy in the continuation is to invest at y^*_{01} for a continuation payoff of $F^*(0,0,y)$. If it chooses to deviate with intensity $s'(0,0,y) = \lambda \in (0,1]$, its continuation payoff is:

$$\frac{\lambda\left[1 - s_1^g(0,0,y)\right] L(0,0,y) + (1-\lambda)\, s_1^g(0,0,y)\, F^*(0,0,y) + \lambda s_1^g(0,0,y)\, S(0,0,y)}{\lambda - \lambda s_1^g(0,0,y) + s_1^g(0,0,y)}.$$

Substituting for $s_1^g(0,0,y)$, this is equal to $F^*(0,0,y)$. Thus, for any subgame starting at $y \in (y_{00}^p, y_{01}^*)$, both firms are indifferent between all possible choices. At $y = y_{00}^p$ the continuation payoff from the candidate MPE strategies is $F^*(0,0,y_{00}^p) = L(0,0,y_{00}^p)$ as for all possible alternatives. Last, the right partial derivative $\partial_y^+ s_1^f(0,0,y'')$ is strictly positive as required by regularity condition (R_2). For the proof that there is no other equilibrium outcome, we refer the reader to Fudenberg and Tirole (1985, Appendix 1).

#3. See the proof of Lemma 2. ■

Proof of Lemma 3. Let $Y_t = y$. The surplus at date t is the expected cumulative discounted sum of the maximum that consumers are willing to pay for the production of each capacity slice at each future date, minus discounted capacity expenditures:

$$\Phi(y) = E^y \left\{ \int_t^\infty \left[\sum_{j=1}^{j_s} e^{-rs} Y_s D(j) \right] ds - \sum_{i=0}^{\bar{\imath}-1} e^{-r\tau_i} I \right\},$$

where j_s is the number of capacity units in place at s and τ_i is the date of introduction of the $(i+1)\,th$ capacity unit. This may be written:

$$\Phi(y) = E^y \left\{ \sum_{i=0}^{\bar{\imath}-1} \int_{\tau_i}^\infty e^{-rs} Y_s D(i+1)\, ds - e^{-r\tau_i} I \right\}.$$

The rest of the derivation is a mere adaptation of the proof of Lemma **1**. ■

Proof of Lemma 4. This is a straightforward adaptation of Lemma 2. ■

Proof of Proposition 4. For the proof that there is no other equilibrium outcome, we again refer the reader to Fudenberg and Tirole (1985, Appendix 1).

1. The proof is a mere adaptation from the proof of Proposition 2, for the following equilibrium strategies:

$$s_1^f(1,1,y) = s_1^g(1,1,y) = \begin{cases} 0, \ y \in [0, y_{11}^p) \\ \frac{L(1,1,y) - F^*(1,1,y)}{L(1,1,y) - S(1,1,y)}, \ y \in [y_{11}^p, y_{02}^*) \\ 1, \ y \in [y_{02}^*, \infty) \end{cases}.$$

#2.a. Let $L(1,1,y) \leq S^*(1,1,y)\ \forall y \in (0, y_{12}^*]$. By the definition of $S^*(1,1,y)$, one has $L(1,1,y) \leq S^*(1,1,y)\ \forall y \in (0, y_{11}^{s*}]$. We will show that the following (tacit collusion)

strategies, whose equilibrium payoff is $S^*(1,1,y)$ for both firms, yield a MPE:

$$s_1^f(1,1,y) = s_1^g(1,1,y) = \begin{cases} 0, & y \in [0, y_{11}^{s*}) \\ 1, & y \in [y_{11}^{s*}, \infty) \end{cases},$$
$$s_1^f(1,2,y) = s_1^g(1,2,y) = \begin{cases} 0, & y \in [0, y_{12}^{*}) \\ 1, & y \in [y_{12}^{*}, \infty) \end{cases}.$$

For either firm, say f, a deviation from $s_1^f(1,1,y)$ either results in an investment after y_{11}^{s*} is reached, or in an investment before y_{11}^{s*} is reached. In the former instance, since g has already invested when f invests, the payoff is $F(1,2,y) < F^*(1,2,y) \leq S^*(1,1,y)$ where the last inequality follows from the fact that $y_{11}^s = y_{12}^*$ is admissible in the maximization that defines $S^*(1,1,y)$. If the deviation results in an investment by f before y_{11}^{s*} is reached, then g applies $s_1^g(1,2,y)$. The payoff to f is $L(1,1,y)$ if the deviation occurs before y_{12}^* is reached and $S(1,1,y)$ if it occurs at or after y_{12}^* (since in that case g invests immediately). Since $S(1,1,y) \leq S^*(1,1,y)$, the above strategies yield a MPE with joint investment at y_{11}^{s*}. To complete the proof of existence, we need to show necessity, i.e. that no equilibrium exists if $L(1,1,y) \leq S^*(1,1,y)$ is violated. First consider joint investment at y_{12}^s with payoff $\tilde{S}(1,1,y) \leq S^*(1,1,y)$. Clearly the above strategy adjusted for joint investment at y_{12}^s rather than y_{12}^{s*} yields a MPE if $L(1,1,y) \leq \tilde{S}(1,1,y) \quad \forall y \in (0, y_{12}^s]$; but $L(1,1,y) \leq \tilde{S}(1,1,y) \quad \forall y \in (0, y_{12}^s]$ implies $L(1,1,y) \leq S^*(1,1,y) \ \forall y \in (0, y_{12}^*]$. Second consider any situation with $L(1,1,y) > S^*(1,1,y)$ for some $y \in (0, y_{12}^*]$; this implies $L(1,1,y) > \tilde{S}(1,1,y)$ for any joint investment threshold other than y_{12}^{s*}; then deviation at y is preferable for any candidate joint investment threshold. This completes the proof of existence. As explained in the text, when $\pi_{22} < \pi_{11}$, $y_{11}^{s*} \to \infty$; thus firms never invest. Rents are equal by the definition of S.

#2.d. Shown in the proof of #2.a.. ■

Proof of Proposition 5. Assume that $\pi_{22} - \pi_{11} > 0$: tacit collusion involves investment by both firms rather than inaction so that:[13]

$$S(1,1,y) = \frac{\pi_{11}}{r-\alpha} y + \left(\frac{y}{y_{11}^s}\right)^\beta \left(\frac{\pi_{22} - \pi_{11}}{r-\alpha} y_{11}^s - I\right), \quad y < y_{11}^s.$$

The value of taking the lead at $Y_t = y$ is:

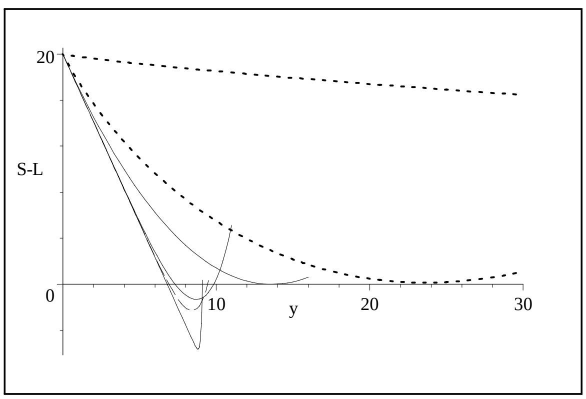
$$L(1,1,y) = \frac{\pi_{21}}{r-\alpha} y - I + \left(\frac{y}{y_{12}^*}\right)^\beta \left(\frac{\pi_{22} - \pi_{21}}{r-\alpha} y_{12}^*\right), \ y = y_{11} < y_{12}^* < y_{11}^s.$$

[13]The opposite case $\pi_{22} - \pi_{11} \leq 0$ can be treated by the same approach, using $S(1,1,y) = \frac{\pi_{11}}{r-\alpha} y$ as the value of tacit collusion by inaction.

By Proposition 4.2.a, a tacit collusion equilibrium exists if $S - L$ is positive for all $y < y_{12}^*$.[14] Thus we study the sign of:

$$\begin{aligned} D(1,1,y) &\equiv S(1,1,y) - L(1,1,y) \\ &= \frac{\pi_{11} - \pi_{21}}{r - \alpha} y + I \\ &\quad + y^\beta \left(\frac{\beta - 1}{\beta I}\right)^{\beta - 1} \left(\beta^{-1} \left(\frac{\pi_{22} - \pi_{11}}{(r - \alpha)}\right)^\beta - \left(\frac{\pi_{22} - \pi_{12}}{r - \alpha}\right)^\beta \left(\frac{\pi_{22} - \pi_{21}}{\pi_{22} - \pi_{12}}\right)\right) \end{aligned}$$

where $y_{12}^* = \frac{r-\alpha}{\pi_{22}-\pi_{12}} I \frac{\beta}{\beta-1}$ and $y_{11}^{s*} = \frac{1}{\pi_{22}-\pi_{11}} (r - \alpha) I \frac{\beta}{\beta-1}$ have been substituted in. $D(1,1,y)$ is positive at $y = 0$. For any market demand parameters (the π_{ij}'s) satisfying Assumption 2 and $\pi_{22} > \pi_{11}$, the negative linear term dominate at small values of y so that D initially diminishes from a positive initial value of I; the term in y^β is positive and, since $\beta > 1$, eventually dominates. Thus $D(1,1,y)$ reaches a minimum with respect to y at some finite positive value of y, which can be shown to be within the admissible interval. In the graph below, the minimum of D is negative for some parameter values (the tacit collusion equilibrium does not exist) and it is positive for other parameters (the tacit collusion equilibrium exists).



$S - L$: (dots: low β; solid: high β)

For any market demand parameters (the π_{ij}'s) satisfying Assumption 2 and $\pi_{22} > \pi_{11}$, we will investigate the effects of changing β, α, and r subject to the restrictions spelled out in the paper: $r > \alpha > 0$ and $\beta > 1$. Changes in σ affect β without affecting any other parameter in $D(1,1,y)$; precisely, as σ increases from zero to infinity, β diminishes from some finite value to a limit of 1. Smaller values of β correspond to the dotted D curves on the graph, which remain positive at any value of y. To prove this result we study the limit of D as $\sigma \to \infty$:

$$\lim_{\beta \downarrow 1} \left(\frac{\pi_{11}-\pi_{21}}{r-\alpha} y + I + y^\beta \left(\frac{\beta-1}{\beta I}\right)^{\beta-1} \left(\beta^{-1} \left(\frac{\pi_{22}-\pi_{11}}{(r-\alpha)}\right)^\beta + \left(\frac{\pi_{22}-\pi_{12}}{r-\alpha}\right)^\beta \left(\frac{\pi_{21}-\pi_{22}}{\pi_{22}-\pi_{12}}\right)\right)\right)$$

[14] For $y_{12}^* < y \leq y_{11}^s \leq y_{11}^{s*}$, $S(1,1,y)$ is higher than the continuation of $L(1,1,y)$.

$$= \frac{\pi_{11}-\pi_{21}}{r-\alpha} y + I + \lim_{\beta\to 1}\left(y^\beta\left(\frac{\beta-1}{\beta I}\right)^{\beta-1}\left(\beta^{-1}\left(\frac{\pi_{22}-\pi_{11}}{(r-\alpha)}\right)^\beta + \left(\frac{\pi_{22}-\pi_{12}}{r-\alpha}\right)^\beta\left(\frac{\pi_{21}-\pi_{22}}{\pi_{22}-\pi_{12}}\right)\right)\right)$$
$$= \frac{\pi_{11}-\pi_{21}}{r-\alpha} y + I - \frac{\pi_{11}-\pi_{21}}{r-\alpha} y = I.$$

Although $\beta = 1$ is not an admissible value, any $\beta > 1$ is admissible. By continuity of D in β, there exist some $\varepsilon > 0$ such that, for $\beta = 1+\varepsilon$, D is positive for any y. This proves #1.

A similar approach is used to prove #2 and #3, allowing for the fact that changes in r or α not only affect β, but also other terms in D. Noting that $\lim_{r\downarrow\alpha}\beta = 1^+$, we study the limit of D as $r \downarrow \alpha$:

$$\lim_{r\downarrow\alpha}\left(\frac{\pi_{11}-\pi_{21}}{r-\alpha} y + I + y^\beta\left(\frac{\beta-1}{\beta I}\right)^{\beta-1}\left(\beta^{-1}\left(\frac{\pi_{22}-\pi_{11}}{(r-\alpha)}\right)^\beta + \left(\frac{\pi_{22}-\pi_{12}}{r-\alpha}\right)^\beta\left(\frac{\pi_{21}-\pi_{22}}{\pi_{22}-\pi_{12}}\right)\right)\right)$$
$$= \lim_{r\downarrow\alpha}\left\{\frac{\pi_{11}-\pi_{21}}{r-\alpha} y + I + \lim_{\beta\downarrow 1}\left(y^\beta\left(\frac{\beta-1}{\beta I}\right)^{\beta-1}\left(\beta^{-1}\left(\frac{\pi_{22}-\pi_{11}}{(r-\alpha)}\right)^\beta + \left(\frac{\pi_{22}-\pi_{12}}{r-\alpha}\right)^\beta\left(\frac{\pi_{21}-\pi_{22}}{\pi_{22}-\pi_{12}}\right)\right)\right)\right\}$$
$$= \lim_{r\downarrow\alpha}\left\{\frac{\pi_{11}-\pi_{21}}{r-\alpha} y + I - \frac{\pi_{11}-\pi_{21}}{r-\alpha} y\right\} = I.$$

This implies that D is positive when r is close enough to, while strictly higher than, α, proving #2 and #3. ■

Proof of Lemma 5. Let K^c be the smaller integer such that $K^c \geq x^c$; since $\pi_{22} > \pi_{12}$, $K^c \geq 2$ which implies $\pi_{23} > \pi_{13}$. (See Short-run Cournot game in Section 3) ■

Proof of Lemma 6. For $y \leq y^*_{13}$, let $G(2,1,y) \equiv L(2,1,y) - F^*(2,1,y)$ denote the gain for the bigger firm to become leader at the current value y of Y_t:

$$\begin{aligned} G(2,1,y) &= \frac{\pi_{31}}{r-\alpha} y - I + \left(\frac{y}{y^*_{13}}\right)^\beta\left(\frac{\pi_{32}-\pi_{31}}{r-\alpha} y^*_{13}\right) \\ &\quad - \sup_{y_{22}}\left[\frac{\pi_{22}}{r-\alpha} y + \left(\frac{y}{y_{22}}\right)^\beta\left(\frac{\pi_{32}-\pi_{22}}{r-\alpha} y_{22} - I\right)\right], \end{aligned} \tag{A.4}$$

where $y^*_{13} = \frac{1}{\pi_{23}-\pi_{13}}(r-\alpha) I \frac{\beta}{\beta-1}$; $y^*_{22} = \frac{1}{\pi_{32}-\pi_{22}}(r-\alpha) I \frac{\beta}{\beta-1}$. (If the firm never invests as follower, $y^*_{22} \to \infty$, and the last term vanishes). We show that $G(2,1,y^*_{13}) \leq 0$. Substituting y^*_{13} for y and simplifying,

$$G(2,1,y^*_{13}) = -I + \left(\frac{\pi_{32}}{r-\alpha} y^*_{13}\right) - \sup_{y_{22}}\left[\frac{\pi_{22}}{r-\alpha} y^*_{13} + \left(\frac{y^*_{13}}{y_{22}}\right)^\beta\left(\frac{\pi_{32}-\pi_{22}}{r-\alpha} y_{22} - I\right)\right].$$

If the argmax of the last term is y_{13}^*, then $G(2,1,y)=0$; otherwise that term is larger than the first part so that $G(2,1,y_{13}^*)<0$. Since $G(2,1,y)$ is concave and $G(2,1,0)=-I$, if a strictly positive maximum occurs for some $y<y_{13}^*$, then there exists two values of y in the interval $[0,y_{13}^*]$ such that $G(2,1,y)=0$; we define y_{12}^p as the smallest root. ■

Proof of Proposition 6. #1. Existence will be shown by construction. By Assumption 3, if the bigger firm takes the lead at $y\leq y_{13}^*$, the sole possible continuation is one where the smaller firm invests at y_{13}^*; its best strategy is then:

$$s_1(1,3,y)=\begin{cases}0, \ y\in[0,y_{13}^*)\\ 1, \ y\in[y_{13}^*,\infty)\end{cases}.$$

Alternatively, if the smaller firm takes the lead at some $y\leq y_{13}^*$, it is a dominant strategy for the bigger firm to follow at some y_{22}^*, $y_{13}^*<y_{22}^*\leq\infty$:

$$s_1(2,2,y)=\begin{cases}0, \ y\in[0,y_{22}^*)\\ 1, \ y\in[y_{22}^*,\infty)\end{cases}.$$

Thus the gain for the bigger firm to become leader, if the alternative is the smaller firm taking the lead, is $G(2,1,y)$.

As far as the smaller firm is concerned, two alternatives may arise. Trivially, if becoming leader is a dominated strategy for the bigger firm ($G(2,1,y)\leq 0\ \forall\ y\leq y_{13}^*$), then the result holds, with the smaller firm investing at its stand-alone date, i.e. when y reaches y_{12}^* for the first time. Alternatively, if $G(2,1,y)>0$ for some values of $y\leq y_{13}^*$ and if the strategy of the bigger firm is to take the lead if the smaller firm does not do so first, then the gain for the smaller firm to become leader immediately is $G(1,2,y)\equiv L(1,2,y)-F^*(1,2,y)$, $y\leq y_{13}^*$:

$$G(1,2,y)=\frac{\pi_{22}-\pi_{13}}{r-\alpha}y-I+\left(\frac{y}{y_{22}^*}\right)^{\beta}\left(\frac{\pi_{23}-\pi_{22}}{\pi_{32}-\pi_{22}}\frac{\beta}{\beta-1}\right)I-\left(\frac{y}{y_{13}^*}\right)^{\beta}\frac{1}{\beta-1}I$$

We shall compare $G(2,1,y)$, the gain for the bigger firm to become leader immediately, as given by $(A.4)$, with $G(1,2,y)$, the gain for the smaller firm to become leader immediately. Thus, for $y \leq y_{13}^{*}$,

$$
\begin{aligned}
G(1,2,y) - G(2,1,y) &= \frac{2\pi_{22} - \pi_{13} - \pi_{31}}{r-\alpha} y \\
&+ \left(\frac{y}{y_{22}^{*}}\right)^{\beta} \left(\frac{\pi_{23} - \pi_{22}}{\pi_{32} - \pi_{22}}\beta + 1\right) \frac{I}{\beta - 1} - \left(\frac{y}{y_{13}^{*}}\right)^{\beta} \left(1 + \frac{\pi_{32} - \pi_{31}}{\pi_{23} - \pi_{13}}\beta\right) \frac{I}{\beta - 1} \\
= &\left[\left(\frac{y}{y_{13}^{*}}\right)^{\beta} \frac{\beta\pi_{31} - \beta\pi_{32} + \pi_{23} - \pi_{13}}{\pi_{23} - \pi_{13}} - \left(\frac{y}{y_{22}^{*}}\right)^{\beta} \frac{\beta\pi_{22} - \beta\pi_{23} + \pi_{32} - \pi_{22}}{\pi_{32} - \pi_{22}}\right] \frac{\beta I}{\beta - 1}.
\end{aligned}
$$

If this expression is positive for some y, it is positive for any y. Take $y = y_{13}^{*}$; we look for the sign of $S = \left(\frac{\beta\pi_{31} - \beta\pi_{32} + \pi_{32} - \pi_{22}}{\pi_{23} - \pi_{13}}\right) - \left(\frac{y_{13}^{*}}{y_{22}^{*}}\right)^{\beta} \left(\frac{\beta\pi_{22} - \beta\pi_{23} + \pi_{23} - \pi_{13}}{\pi_{32} - \pi_{22}}\right)$. Since the second term in S is negative (minus a positive one), and $\left(\frac{y_{13}^{*}}{y_{22}^{*}}\right)^{\beta} = \left(\frac{\pi_{32} - \pi_{22}}{\pi_{23} - \pi_{13}}\right)^{\beta} \leq \left(\frac{\pi_{32} - \pi_{22}}{\pi_{23} - \pi_{13}}\right) < 1$,

$$
S > \frac{\beta\pi_{31} - \beta\pi_{32} + \pi_{32} - \pi_{22}}{\pi_{23} - \pi_{13}} - \left(\frac{\pi_{32} - \pi_{22}}{\pi_{23} - \pi_{13}}\right) \frac{\beta\pi_{22} - \beta\pi_{23} + \pi_{23} - \pi_{13}}{\pi_{32} - \pi_{22}}
$$

Thus the sign of S is the same as the sign of:

$$
\begin{aligned}
& \beta\pi_{31} - \beta\pi_{32} + \pi_{32} - \pi_{22} - (\beta\pi_{22} - \beta\pi_{23} + \pi_{23} - \pi_{13}) \\
= \; & p_5 - p_4 - \beta(p_5 - p_4) > 0 \text{ since } \beta > 1 \text{ and } p_5 - p_4 < 0,
\end{aligned}
$$

where $p_i = D^{-1}(i)$. Thus the gain from becoming leader is higher for the small firm than it is for the bigger firm at any $y < y_{13}^{*}$. For any y such that $G(2,1,y) \geq 0$, the best response for the small firm to a strategy by the bigger firm of taking the lead at $Y_t = y$ is to preempt at $y - \varepsilon$. Consequently a preemption equilibrium with the bigger firm as leader does not exist.

Consider preemption by the smaller firm. By assumption $G(2,1,y) > 0$ for some $y < y_{13}^{*}$ so that the bigger firm may invest first if the smaller one does not preempt. By Lemma 6, $G(2,1,y_{12}^{p}) = 0$ so that, since $G(1,2,y) - G(2,1,y) > 0$, $G(1,2,y_{12}^{p}) > 0$. Then the smaller firm should invest at $\min\{y_{12}^{p}, y_{12}^{*}\}$, which is achieved in equilibrium for the following strategies (note the smaller firm takes the lead with probability one):

$$
\begin{aligned}
s_1(1,2,y) &= \begin{cases} 0, \; y \in [0, \min\{y_{12}^{p}, y_{12}^{*}\}) \\ 1, \; y \in [\min\{y_{12}^{p}, y_{12}^{*}\}, \infty) \end{cases}, \\
s_1(2,1,y) &= \begin{cases} 0, \; y \in [0, \min\{y_{12}^{p}, y_{12}^{*}\}) \\ \frac{L(2,1,y) - F^{*}(2,1,y)}{L(2,1,y) - S(2,1,y)}, \; y \in [\min\{y_{12}^{p}, y_{12}^{*}\}, y_{02}^{*}) \\ 1, \; y \in [y_{02}^{*}, \infty) \end{cases}.
\end{aligned}
$$

The rest of the proof is a mere adaptation of the proof of Proposition 2. For the proof of uniqueness, we refer the reader to Fudenberg and Tirole (1985, Appendix 1).

#2. Shown in 1. ■

Proof of Proposition 7. (tacit collusion with different capacities) Under Assumption 3:

#1. If $\pi_{32} - \pi_{22} \leq 0$, as shown in the main text (see definition of $F(2,1,y)$ and y^*_{21}), the bigger firm does not find it profitable to invest if the smaller one has taken the lead; thus no collusion MPE exists. ■

#2. $\pi_{32} - \pi_{22} > 0$. By adapting the proof of Proposition 4 one can show that collusion is an MPE if and only if $S(2,1,y) - L(2,1,y) \geq 0$ and $S(1,2,y) - L(1,2,y) \geq 0$ for all $y \leq y^s_{21} = y^s_{12}$ where the last equality holds because both firms must invest simultaneously. Thus we compute $S - L$ for both firms:

$$\begin{aligned} & S(2,1,y) - L(2,1,y) \\ = \; & \frac{\pi_{21} - \pi_{31}}{r - \alpha} y + I \\ & + y^\beta \left(\frac{\beta - 1}{\beta I}\right)^{\beta - 1} \left(\left(\frac{\pi_{23} - \pi_{12}}{(r-\alpha)}\right)^\beta \frac{\pi_{32} - \pi_{21} - \frac{\beta-1}{\beta}(\pi_{23} - \pi_{12})}{\pi_{23} - \pi_{12}} + \left(\frac{\pi_{23} - \pi_{13}}{r - \alpha}\right)^\beta \frac{\pi_{31} - \pi_{32}}{\pi_{23} - \pi_{13}} \right) \end{aligned}$$

and:

$$\begin{aligned} S(1,2,y) - L(1,2,y) \; = \; & \frac{\pi_{21} - \pi_{31}}{r - \alpha} y + I \\ & + y^\beta \left(\frac{\beta - 1}{\beta I}\right)^{\beta - 1} \left(\left(\frac{\pi_{23} - \pi_{12}}{(r-\alpha)}\right)^\beta \beta^{-1} + \left(\frac{\pi_{32} - \pi_{22}}{(r - \alpha)}\right)^\beta \left(\frac{\pi_{22} - \pi_{23}}{\pi_{32} - \pi_{22}}\right) \right) \end{aligned}$$

Some calculations show that $y^*_{13} = \frac{1}{\pi_{23} - \pi_{13}}(r-\alpha) I \frac{\beta}{\beta - 1}$; $y^*_{22} = \frac{1}{\pi_{32} - \pi_{22}}(r-\alpha) I \frac{\beta}{\beta-1}$. Let y^{s*}_{12} and y^{s*}_{21} be the values of y that maximize $S(1,2,y)$ and $S(2,1,y)$ respectively. That is, $y^{s*}_{12} = \frac{1}{\pi_{23} - \pi_{12}}(r-\alpha) I \frac{\beta}{\beta-1}$; $y^{s*}_{21} = \frac{1}{\pi_{32} - \pi_{21}}(r - \alpha) I \frac{\beta}{\beta - 1}$ with $y^*_{13} < y^*_{22} < \min(y^{s*}_{12}, y^{s*}_{21}) = y^{s*}_{12}$. Since S is decreasing in y beyond its maximum, it is a dominant strategy for one at least of the firms to invest when $y \geq y^{s*}_{12}$. Thus we take $y^s_{12} = y^{s*}_{12}$ and look for conditions under which both $S(1,2,y) - L(1,2,y)$ and $S(2,1,y) - L(2,1,y)$ are non negative for any $y \leq y^s_{12}$. The rest of the proof is otherwise similar to that of Proposition 4 #2. ■

#3. It can be verified that the value that maximizes $S(1,2,y) + S(2,1,y)$ is higher than $\min(y^{s*}_{12}, y^{s*}_{21})$. ■